Rendering: Materials 1

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Today's Roadmap

- Adding refractions
  - Snell's Law
  - Fresnel Reflectance
  - Specular BTDF

- Important concepts
  - Chromatic Aberration
  - Heckbert Notation
  - Caustics

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Reflection Model Sources

- *Physical (wave) optics:*
  - Derived using a detailed model of light
  - Treating it as wave and computing solutions to Maxwell's equations
  - Computationally expensive, usually not appreciably more accurate

- ***Geometric optics***:
  - Requires surface's low-level scattering and geometric properties
  - Closed-form reflection models derived from these properties
  - More tractable, complex wave effects like polarization are ignored

The Missing Part of the Rendering Equation

$$L_e(x,v) = E(x,v) + \int_\Omega \boxed{f_r(x,\omega \to v)} L_i(x,\omega)\cos(\theta_x)\,\mathrm{d}\omega$$

- Bidirectional Scattering Distribution Function (BSDF)
- Describes the light transport properties of the material
- So far, we only implemented simple diffuse surface reflections
- Can model reflections, refractions, volumetric scattering…

Bidirectional Reflectance Distribution Function (BRDF)

- Considers only the **reflection** of incoming light onto a surface
  - The BRDF is a limited instance of the full BSDF (e.g., no transparency)
  - Good for starting out, complex materials need full BSDF
  - More on that in another lecture

- A BRDF function $f_r(x, \omega_i \rightarrow \omega_o)$ with input directions ω_i, ω_o
  - uses convention: ω_i and ω_o are assumed to point away from x
  - How much irradiance from ω_i is reflected as radiance to ω_o at x?

Bidirectional Reflectance Distribution Function (BRDF)

- "How much irradiance from ω_i is reflected as radiance to ω_o at x?"

- $f_r(x, \omega_i \rightarrow \omega_o) = \dfrac{dL_i(x, \omega_o)}{dE_i(x, \omega_i)} = \dfrac{dL_i(x, \omega_o)}{L_i(x, \omega_i)\cos_\theta(\omega_i)\, d\omega_i}$

$$L_e(x, v) = E(x, v) + \int_\Omega f_r(x, \omega \rightarrow v) L_i(x, \omega) \cos(\theta_x)\, \mathrm{d}\omega$$

- **Helmholtz reciprocity**: $f_r(x, \omega_i \rightarrow \omega_o) = f_r(x, \omega_o \rightarrow \omega_i)$

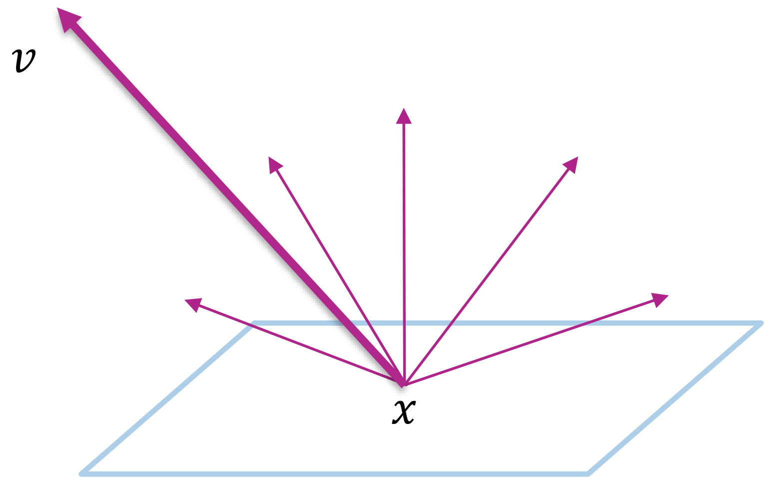
- **Conserves energy**: $\int_\Omega f_r(x, \omega \rightarrow v) \cos\theta\, d\omega \leq 1\ \forall\ v$

Condition for Energy Conservation

- Why must the BRDF f_r fulfill $\int_\Omega f_r(x, \omega \to v) \cos_\theta(\omega)\, d\omega \leq 1$?

- Intuitive interpretation with **reciprocity**: Shine a laser light along $-v$ onto x. We must have $\int_\Omega f_r(x, v \to \omega) \cos_\theta(\omega)\, d\omega \leq 1$

- If we find a direction v for which this is not true, it means we would reflect more light than is coming in



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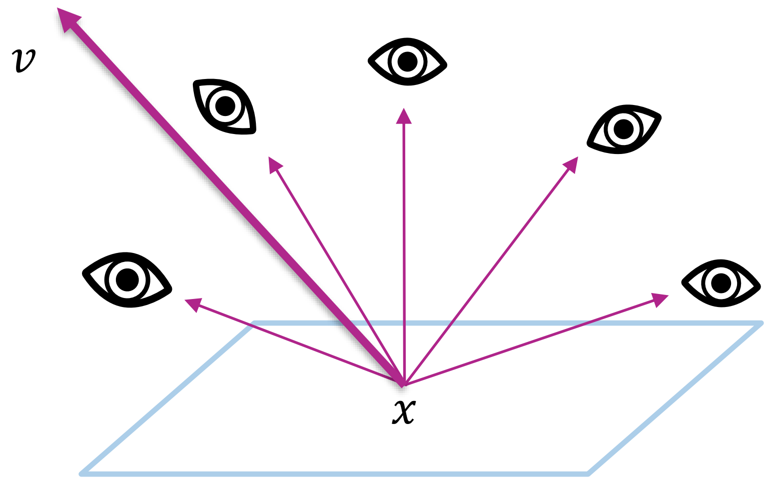
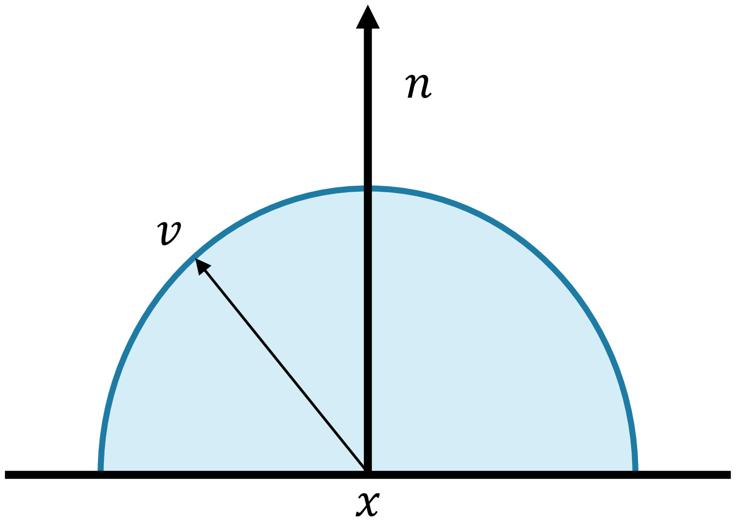
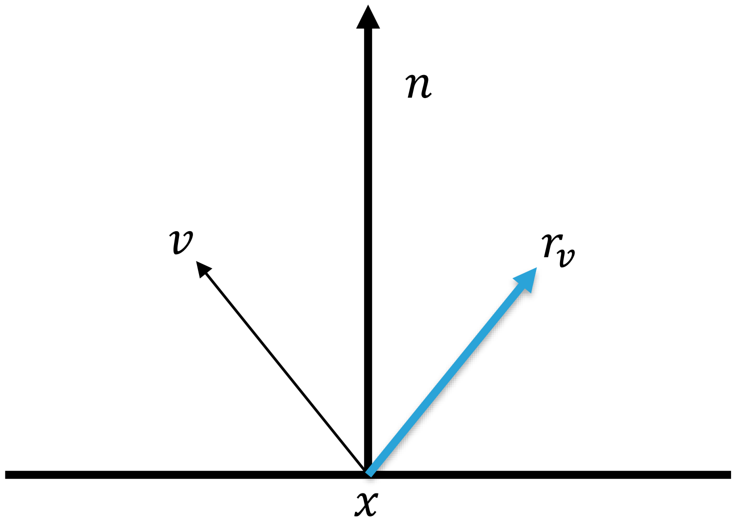


BRDF Types

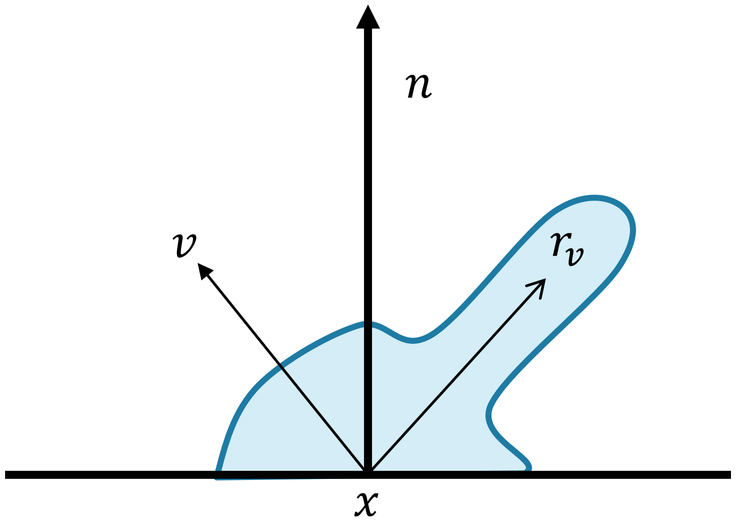
- We usually distinguish three basic BRDF types
  - Perfectly diffuse (light is scattered equally in/from all directions)
  - Perfectly specular (light is reflected in/from exactly one direction)
  - Glossy (mixture of the other two, stronger reflectance around r_v)



Diffuse



Specular



Glossy

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Sampling the BRDF

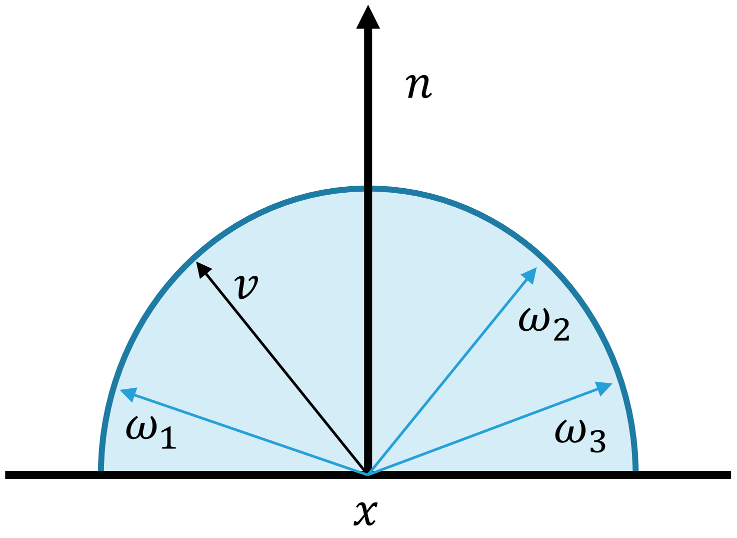
- Before, we considered the BRDF value and sampling of ω separately

- For implementation, it makes a lot of sense to combine them
  - $f_r(x, \omega \rightarrow v)$ depends only on x, v and next ray direction ω
  - Rendering equation: we can't predict L_i, but $f_r(x, \omega \rightarrow v)$ and $\cos\theta$
  - Our renderings will converge faster if the distribution of ω actually matches the shape of $f_r(x, \omega \rightarrow v)\cos\theta$ (**importance sampling**!)
  - If we put the BRDF in charge of choosing our ω, we can make it sample a distribution that directly matches $f_r(x, \omega \rightarrow v)\cos\theta$
  - This actually makes things cleaner in code

How to Handle Diffuse BRDFs

- Diffuse materials reflect same amount of light in/from all directions

- $f_r(x, \omega \rightarrow v) = \frac{\rho}{\pi} \; \forall \; v, \omega \angle n < \frac{\pi}{2}$
  - $\rho =$ amount of reflected light
  - $\rho \leq 1$ in r, g, b

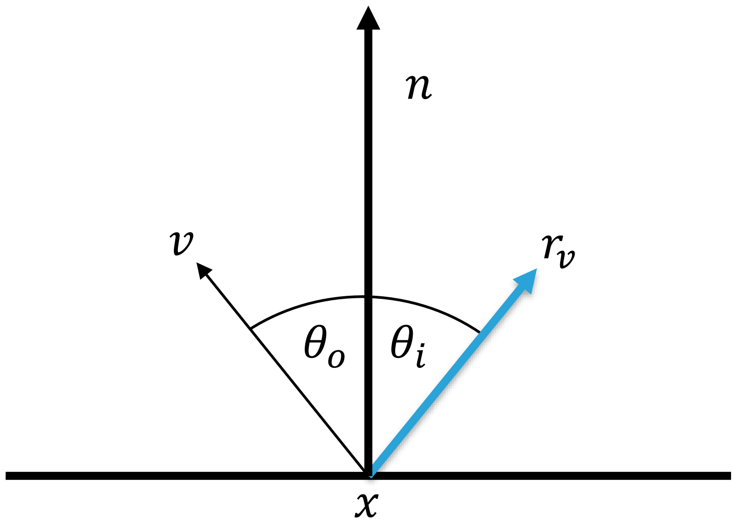


- Importance sampling $f_r(x, \omega \rightarrow v) \cos\theta$ → use $p(\omega) \propto \frac{\rho \cos\theta}{\pi}$
  - Making it a valid PDF leads to $p(\omega) = \frac{\cos\theta}{\pi}$
  - From previous exercise: it's cosine-weighted hemisphere sampling!

Specular Reflection (Mirror)

- The angle of exiting light θ_o is the same as the angle of incidence θ_i

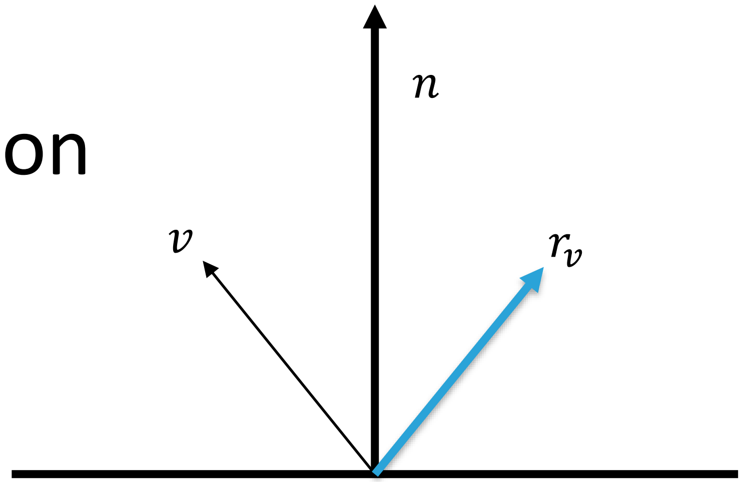
- Incoming light is only transported in a single direction



Specular

How to Handle Specular BRDFs (Mirrors)

- For purely specular BRDFs (a perfect mirror surface), irradiance from the perfect mirror direction r_v is completely reflected to v
- Irradiance coming from any other direction does not reflect at all towards v
- $f_r(x, \omega \to v) > 0 \Leftrightarrow \omega = r_v$
- Problem: if we pick the next direction ω randomly as before, the chances of ever hitting r_v by accident are infinitely small!



The Dirac Delta Function

- Model specular reflection with the Dirac delta function
- Delta function $\delta(x)$ is defined to be 0 everywhere except at $x = 0$
- Use a shifted version $\delta_v(\omega)$ that is 0 everywhere except at $\omega = r_v$
- Per definition, $\int_\Omega \delta_v(\omega)\, d\omega = 1$ to obtain a valid PDF for sampling
- Ponder this for a moment: what value does $\delta_v(r_v)$ have?

Energy-Preserving Specular BRDF

- Full energy preservation: $\int_\Omega f_r(x, \omega \rightarrow v)\, L_i \cos_\theta(\omega)\, d\omega = L_{r_v}$
- If we integrate using $f_r(x, \omega \rightarrow v) = \delta_v(\omega)$, we get $L_{r_v} \cos_\theta(r_v)$
- We lost some light! We compensate: $f_r(x, \omega \rightarrow v) = \frac{\delta_v(\omega)}{\cos_\theta(r_v)}$
- If we consider the properties of the Dirac delta function, we can try to derive the same methods that we used before for diffuse BRDFs

Try to Implement Specular BRDF

- **sample(v)**: mirror v about n (invert v_x, v_y in *local space*) and return
- **evaluate(a, b)**: 0 if $b \neq r_a$, else return $\frac{\delta_a(r_a)}{\cos_\theta(r_a)} = \frac{\infty}{\cos_\theta(r_a)}$
  - **Problem**: How to calculate anything reasonable with ∞?
  - **Problem**: we are comparing two vectors with floats (Stability?)
- **pdf(ω)**: 0 if $\omega \neq r_v$, else: $\delta_v(r_v) = \infty$
- But, if $\omega = r_v$, **evaluate(v, ω) / pdf(ω)** $= \frac{\delta_v(\omega)}{\delta_v(\omega)\cos_\theta(r_v)} = \frac{1}{\cos_\theta(r_v)}$

Revising the Specular BRDF Implementation

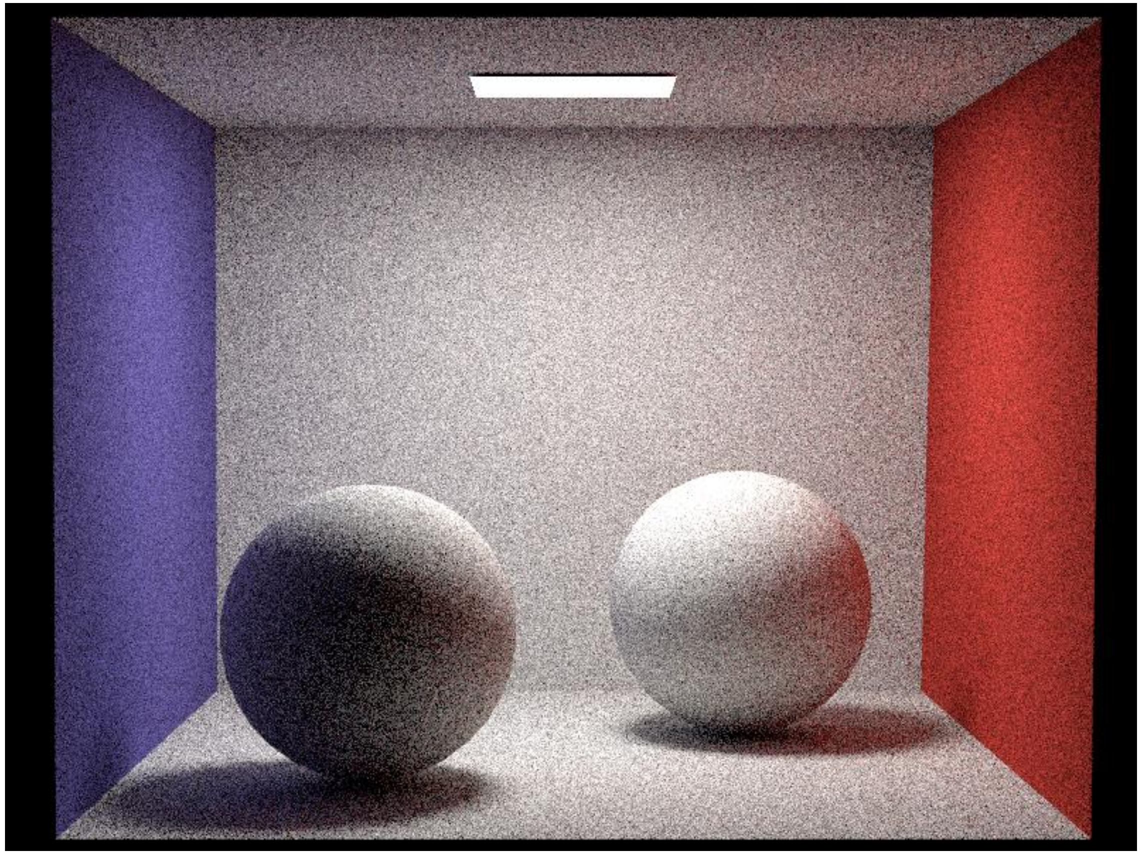
- **sample(v)**: mirror v about n (invert v_x, v_y in *local space*)
  - Return r_v as generated sample direction
  - Return multiplier for L_i as 1 (full radiance passed on)

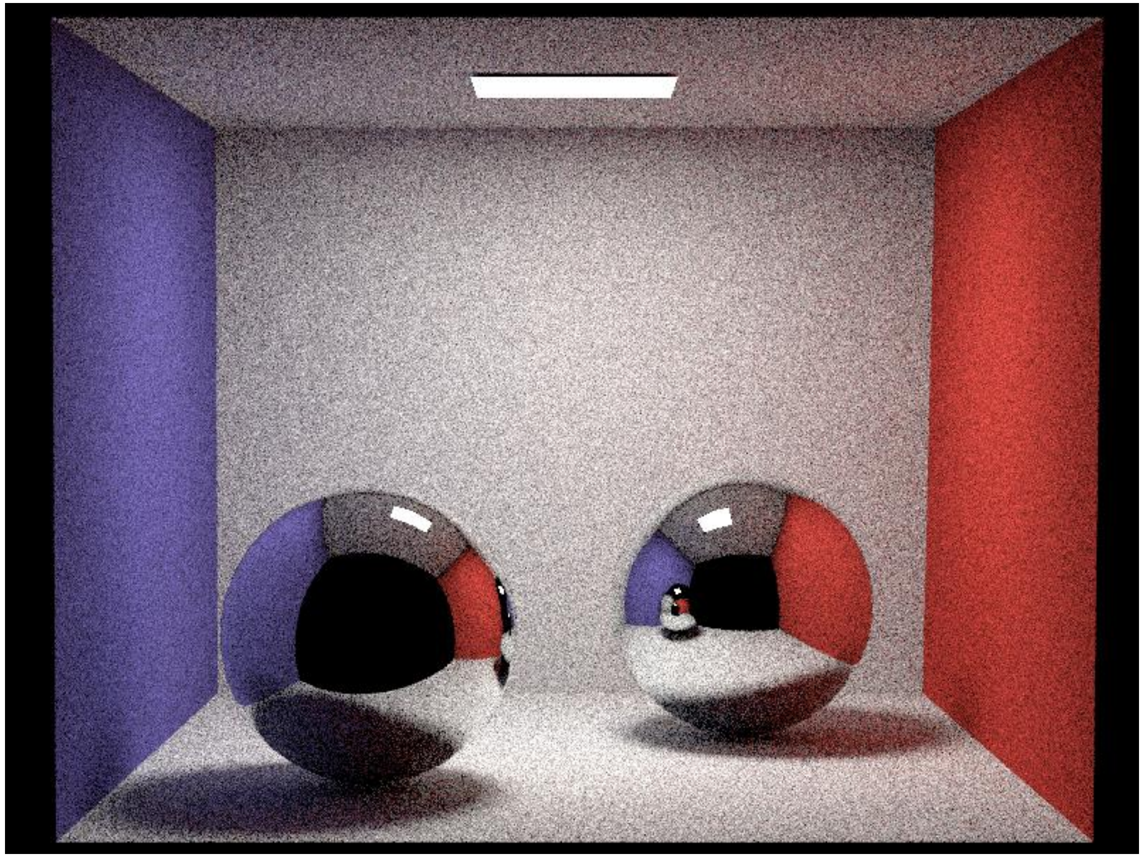
- No other function except **sample** should be able to just *guess* r_v

- **evaluate(a, b)**: always return 0

- **pdf(ω)**: always return 0

Diffuse vs Specular in Action





Specular Reflection and Transmission

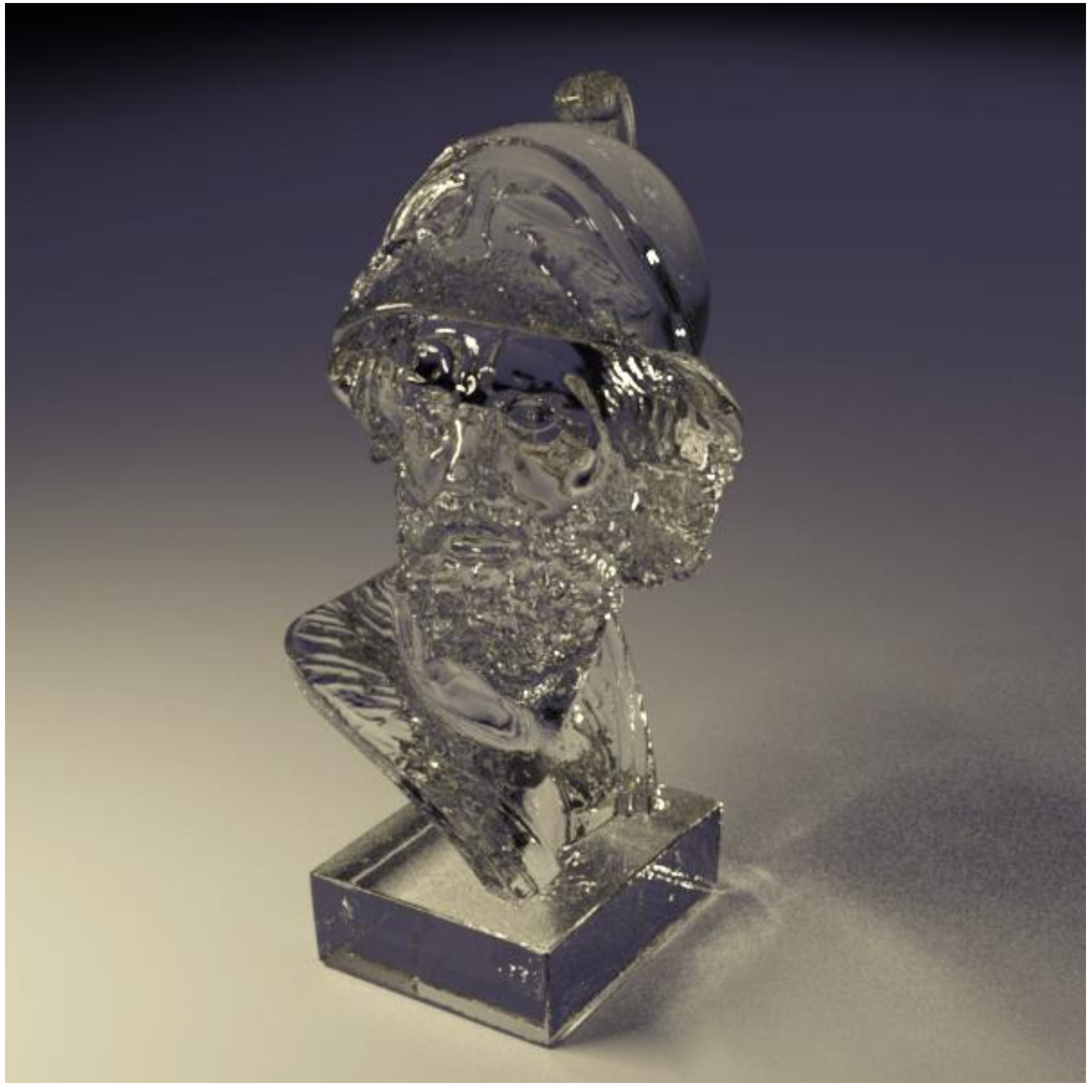
- Before, we assumed that the entire radiance is reflected (mirror)

- This is usually not the case
  - Some light is reflected on the surface
  - Some enters the new material (scattered, absorbed or **refracted**)
  - Meeting point of two different media is called **interface**

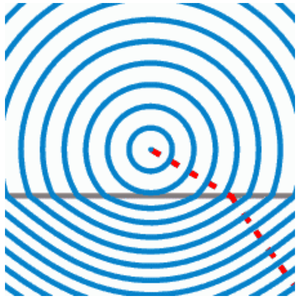
- When entering a different medium, light often changes direction

- Governed by the materials' index of refraction and *Snell's law*

Specular Reflection and Transmission



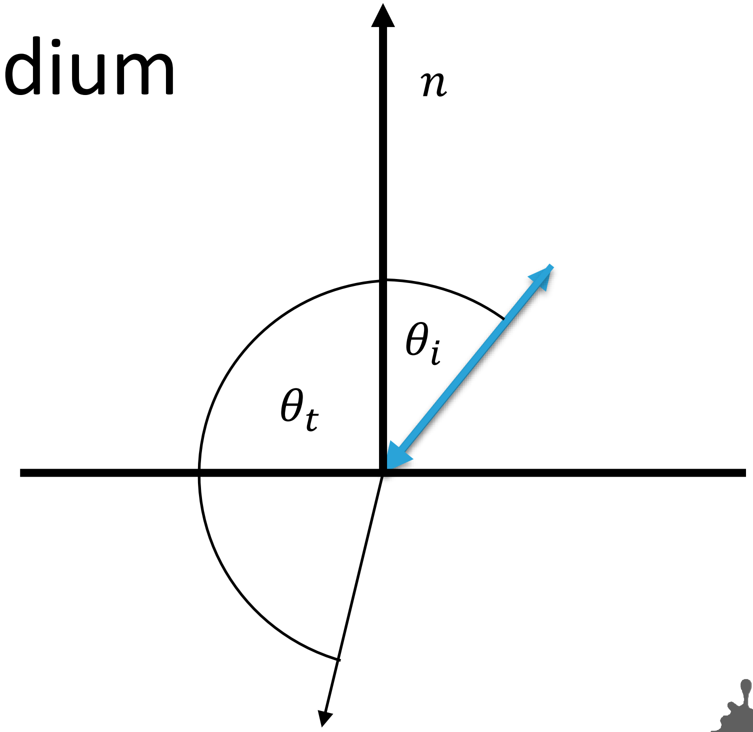
- Based on the indices of refraction for the two materials
  - η_i for the medium that the light ray is currently in
  - η_t for the new medium into which light is transmitted



Public domain, Oleg Alexandrov, Snell's law wavefrons, Wikipedia, "Snell's law"

- Index of refraction: how fast light travels in medium
- Snell's law, essentially:

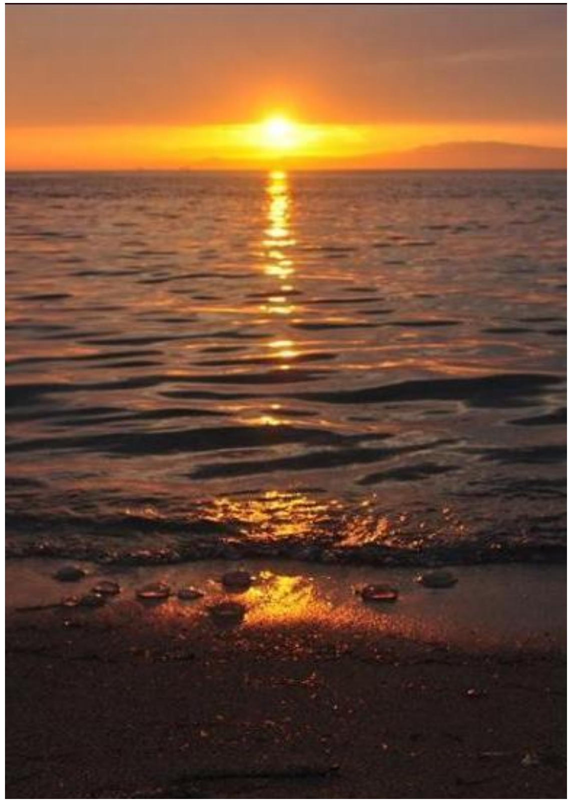
$$\eta_i \sin \theta_i = \eta_t \sin \theta_t$$



- Given η_i, θ_i and η_t, we can easily solve for θ_t

Fresnel Reflectance

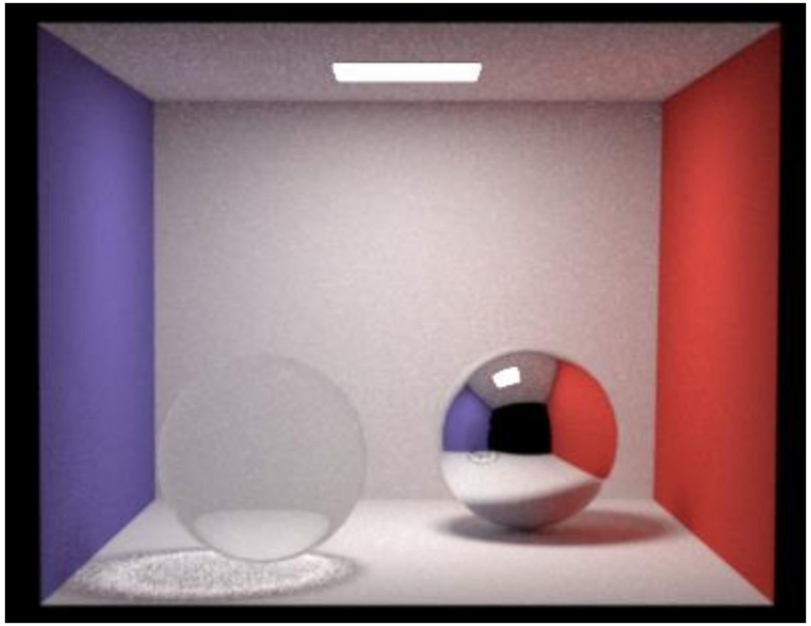
- How much of the light do we reflect?
- Not constant, but actually depends on the θ_i
- The larger θ_i, the better the chance for reflection
- If $\eta_i > \eta_t$, if incident light exceeds a certain θ_i, all light may be reflected (*total internal reflection*)



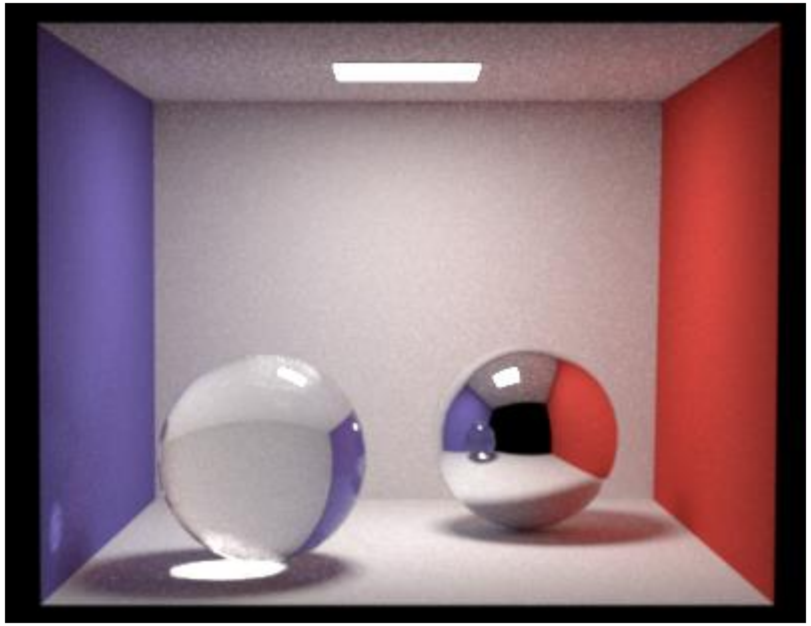
Fresnel Reflectance

- Should be handled differently, depending on the materials involved
- Distinguish how material responds to energy transported by light
- We usually consider three major groups:
  - Dielectrics conduct electricity poorly (glass, air…)
  - Conductors (*metals*, reflect a lot, transmitted light quickly absorbed)
  - Semiconductors (complex, but also rare – we can ignore them)
- Remember, we are aiming for **dielectrics** today

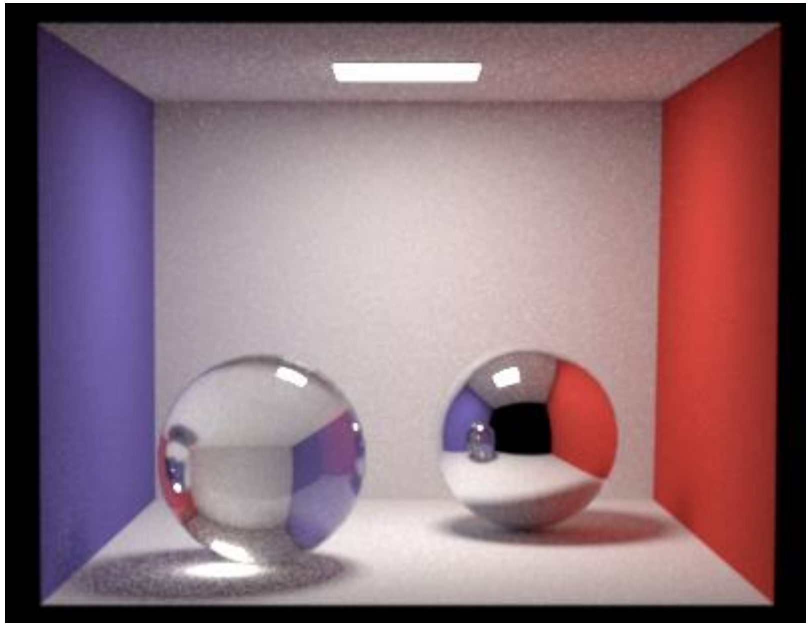
Examples for the Index of Refraction in Dielectrics



$\eta_t = 1.025$ (liquid helium)



$\eta_t = 1.5$ (glass)



$\eta_t = 2.5$ (diamond)

- Gases: 1 – 1.0005 (no-man‘s land from 1.05 to 1.25)
- Liquids: 1.3 (water) – 1.5 (olive oil)
- Solids: 1.3 (ice) – 2.5 (diamond)

Fresnel Reflectance for Dielectrics

- Defined for parallel and perpendicular polarized light ($r_\parallel$ and $r_\perp$):

$$r_\parallel = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}, r_\perp = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t}$$

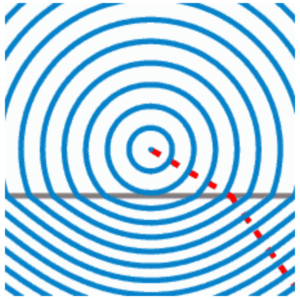
- Amount of **reflected** light (unpolarized light, average of squares):

$$F_r = \frac{1}{2}(r_\parallel^2 + r_\perp^2)$$

- Amount of **refracted** light (conservation of energy): $1 - F_r$

Bidirectional Transmittance Distribution Function (BTDF)

- Refracted light usually changes direction in new medium
  - Remember that we work with radiance: $d\Phi = L_i dA_{\perp} d\omega$
  - Refracted light changes direction → influences radiance!
  - Relate incoming to refracted light:



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$$L_o \cos\theta_o \, dA \sin\theta_o \, d\theta_o \, d\phi_o = (1 - F_r) L_i \cos\theta_i \, dA \sin\theta_i \, d\theta_i \, d\phi_i$$

- Differentiating Snell's law w.r.t. θ, we get:

$$\eta_o \cos\theta_o \, d\theta_o = \eta_i \cos\theta_i \, d\theta_i \rightarrow \frac{\cos\theta_o d\theta_o}{\cos\theta_i d\theta_i} = \frac{\eta_i}{\eta_o}$$

Bidirectional Transmittance Distribution Function (BTDF)

- Substituting, we get:

$$L_o \eta_i^2 d\phi_o = (1 - F_r) L_i \eta_o^2 \, d\phi_i \rightarrow L_o = (1 - F_r) \frac{\eta_o^2}{\eta_i^2} L_i$$

- We have all the required information for the specular BTDF!
  - Use $T(\omega, n)$ to compute direction of ω when refracted at interface
  - Like specular BRDF, light only goes in a single direction
  - Can reuse BRDF $\delta(\omega)$ and normalization (similar implementation!)

$$f_r(x, \omega_i \rightarrow \omega_o) = \frac{\eta_o^2}{\eta_i^2} (1 - F_r) \frac{\delta(\omega_i - T(\omega_o, n))}{|\cos \theta_i|}$$

Bidirectional Transmittance Distribution Function (BTDF)

- When light refracts into a material with a higher η, the energy is compressed into a smaller set of angles

- For the BTDF, $f_r(x, \omega_i \rightarrow \omega_o) = f_r(x, \omega_o \rightarrow \omega_i)$ is not guaranteed
- No reciprocity, but $\eta_i^2 f_r(x, \omega_i \rightarrow \omega_o) = \eta_o^2 f_r(x, \omega_o \rightarrow \omega_i)$ holds!

- If you follow a **view ray**, do the same computations as above, just:
  - Make sure you choose η_i for medium ray comes from
  - Make sure you choose η_t for medium ray goes to

Dielectrics Implementation

- Just continue one path, use Fresnel to decide → reflect or refract?
- View ray behaves exactly like **incident light** in the above equations
- You may find it easier to flip the normal if light **exits** a medium
  - Light that enters e.g. a glass body must also exit at some point
  - I.e., the incoming light ray is not in same hemisphere as n
  - Consistent with using η_i and η_t for current/new medium
- Solving for θ_t, you may get "$\sin\theta_t > 1$" → **total internal reflection**

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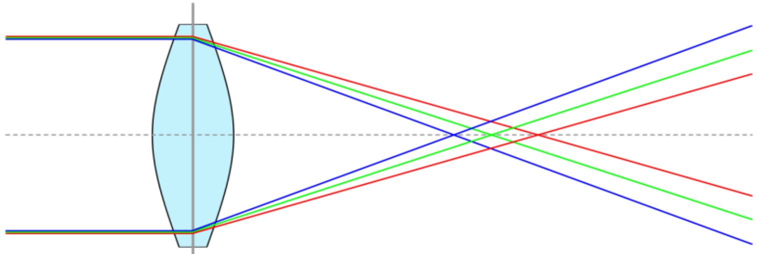
Chromatic Aberration

- Physically speaking, the change in direction is wavelength-dependent
- For proper simulation, would have to at least bend R/G/B differently
- Would spawn two additional rays!
- Can of course be done, but is often ignored (tiny effect on most images)



Public domain, Andreas 06, Chromatic aberration convex, Wikipedia, “Chromatic aberration”

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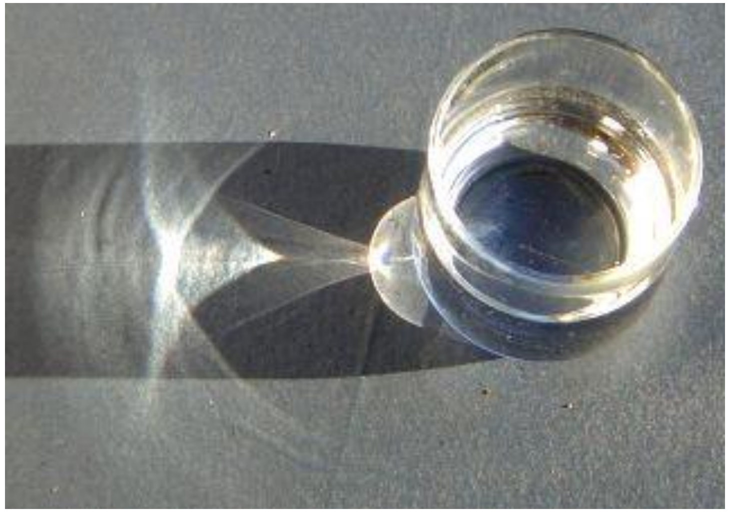
Heckbert Path Notation

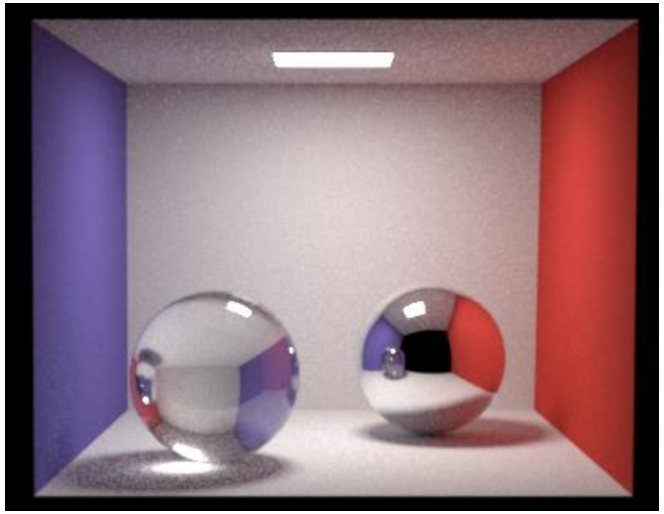
- Assign a letter to every interaction of a light path from light to eye
  - L – light
  - D – diffuse surface
  - S – specular surface
  - E – eye

- Use regex to describe specific (e.g., very challenging) path types
  - LE: direct path from light to eye
  - L(D|S)*E: any path from light to eye
  - LDS+E: a path with one diffuse bounce, followed by specular bounces

A Quick Word on Caustics

- General: focused light from interacting with curved, specular surface





- For us, who are concerned with rendering and path tracing: LS+DE
- Usually challenging to render (takes extremely long to converge)

Video Suggestions

- SIGGRAPH University - Introduction to "Physically Based Shading in Theory and Practice" by Naty Hoffman (!!!)

- SIGGRAPH University - Recent Advances in Physically Based Shading
by Naty Hoffman
(advanced, in the same video there are also some other talks)

References and Further Reading

- *Material for Dielectrics largely based on "Physically Based Rendering" book, chapter 8: Reflection Models*

- [1] Physically Based Rendering (course book, chapters 8 and 9 for materials, chapter 11 for volume rendering)
- [2] Background: Physics and Math of Shading by Naty Hoffman
- [3] Wojciech Jarosz, "Efficient Monte Carlo Methods for Light Transport in Scattering Media", PhD Thesis, https://cs.dartmouth.edu/~wjarosz/publications/dissertation/
- [4] Production Volume Rendering (SIGGRAPH 2017 Course)
- [5] Monte Carlo methods for physically based volume rendering (SIGGRAPH 2018 Course)